



PERTH MODERN SCHOOL
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Independent Public School

Course Mathematics Specialist Year 11

Student name: _____ Teacher name: _____

Date: Monday 23rd March

Task type: Investigation

Time allowed for this task: 45 mins

Number of questions: 4

Materials required: Calculator with CAS capability (to be provided by the student)

Standard items: Pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: Drawing instruments, templates, notes on one unfolded sheet of A4 paper, and up to three calculators approved for use in the WACE Examinations.

Marks available: 35 marks

Task weighting: 10%

Formula sheet provided: Yes

Note: All part questions worth more than 2 marks require working to obtain full marks.

Robots O, X and Y are playing a game using a row of 9 boxes:



The three robots take turns to randomly add their symbol to an empty box, until the grid is complete, e.g.

O	Y	Y	X	O	O	Y	X	X
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They use a scoring system where each row of 3 identical symbols in a completed grid scores a point for that robot.

- [8 = 2 + 2 + 2 + 2 marks]
 - a) Explain why the total number of distinct completed grids is $\binom{9}{3} \times \binom{6}{3} \times \binom{3}{3}$, and evaluate this expression.
 - b) Calculate each of the following probabilities, showing your working.
 - i. Robots X, Y and O each score 1 point after a game.

The three robots now decide to play using a 3x3 grid. Once again, they take turns to add their own symbol **randomly** to one of the empty spaces on the grid, until a completed grid is obtained. They use all nine squares, so that a completed grid always contains 3 of each symbol, e.g.

O	X	Y
X	O	Y
Y	X	O

In a completed grid, each row of 3 identical symbols (horizontal, vertical or diagonal) scores a point for that player. E.g. in the grid above, Robot O scores 1 point and Robots X and Y score 0 points.

2. [16 = 2 + 2 + 2 + 2 + 2 + 3 + 3 marks]

a) Calculate the total number of distinct completed grids.

b) Calculate each of the following probabilities, showing your working.

i. Robots X, Y and O each score 1 point after a game.

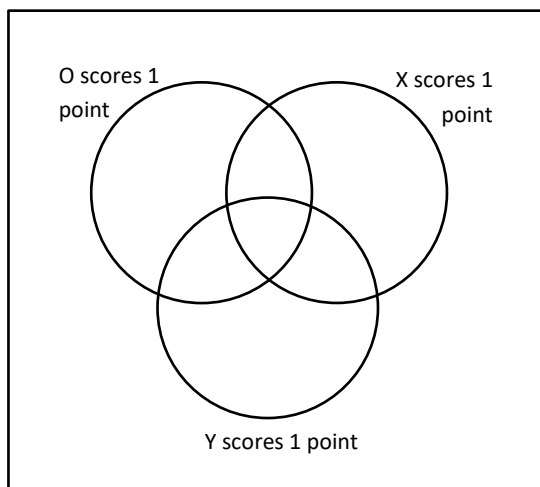
ii. Robots X and Y each score 1 point, but Robot O scores 0 points after a game.

iii. Robot Y scores 1 point (and the other robots may or may not score points).

c) Given your answer to part b (ii), what is the probability that Robots Y and O each score 1 point after a game, but Robot X scores 0? Explain your answer.

d) Calculate the probability that Robot Y scores 1 point after a game, but Robots X and O both score 0 points.

[Hint: the Venn diagram below may be useful.]



e) Calculate the probability that Robots O, X and Y each score 0 points after a game.

Robot Y now has to go, leaving Robots X and O to play together once again.

They decide to play a new game using a 4x4 grid. The rules are exactly the same as before (they take turns to randomly add their symbol), except that now each row of **4** identical symbols (horizontal, vertical or diagonal) in a completed grid scores 1 point for that player (a row of only 3 identical symbols does not score any points).

E.g.

X	O	O	O
X	X	O	X
X	O	X	O
X	O	O	X

 scores a total of 2 points for Robot X (because there is both a vertical and a diagonal row of 4 Xs) and 0 points for Robot O.

3. [11 = 2 + 3 + 4 + 2 marks]

a) Calculate the total number of possible completed grids, showing working.

b) Calculate the probability that Robot X and Robot O each score exactly 2 points after a game. Do not simplify your answer.

c) Calculate the probability that Robot X scores 2 points and Robot O scores 0 points after a game. Do not simplify your answer.

d) Calculate the probability that both robots score at most 1 point after a game.
[Hint: use a Venn diagram.]